

Actions for Curved Branes

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Abstract

The nondeterminantal forms of the Born-Infeld and related brane actions in which the gauge fields couple to both an induced metric and an intrinsic metric are generalised by letting either or both metrics be dynamical. The resulting actions describe ‘brane world’ and cosmological scenarios in which the gauge fields are confined to the brane, while gravity propagates in both the world-volume and the bulk. In particular, for actions involving a nonsymmetric ‘metric’, nonsymmetric gravity propagates on the worldvolume. For 3-branes with a symmetric metric, conformal (Weyl) gravity propagates on the worldvolume and has conformally invariant couplings to the gauge fields.

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1 Introduction

Recently, there has been considerable interest in cosmological and elementary particle physics models in which the usual particles and interactions of the Standard Model are confined to an effectively $(3+1)$ -dimensional hypersurface embedded in a higher-dimensional spacetime [1, 2]. In such models, gravity is allowed to propagate in the bulk of the dimensions transverse to the hypersurface, so that the hypersurface must also be dynamical. Although there are various ways of implementing such scenarios, including those in which the gravitational and gauge interactions are unified at the electroweak scale and there are large compact dimensions [3], string theory provides attractive constructions via D -branes [4, 5, 6]. An apparently different scenario was proposed in [7], where it is argued that the so-called warped compactifications provide phenomenologically attractive alternatives to compactification manifolds. In the construction of [7], space-time is taken to be an AdS_5 space with the near-boundary region cut away and replaced with a wall of constant extrinsic curvature. It is then found that there is a normalizable graviton zero mode in the four dimensional boundary. In [8] (and references quoted therein), it has been suggested on the basis of the AdS/CFT correspondence [9, 10, 11] that this construction amounts to a coupling of gravity to a certain boundary conformal field theory. However, it remains to be seen whether this setup can be embedded in string theory.

In the present paper, we will propose an alternative approach to the construction of brane world models with dynamical worldvolume gravity interacting with gauge and other matter fields. This utilizes the nondeterminantal forms of the Born-Infeld and related Dirac-Born-Infeld actions for p -branes found in refs. [12, 13]. In the particular case of the 3-brane, the action found in [13] involves an intrinsic metric and has a Weyl invariance under rescalings of the metric. Since there are several reasons for imposing scale invariance on the brane world action [14], we seek a coupling to worldvolume gravity which preserves this symmetry at the classical level. This is achieved by promoting the intrinsic metric to a dynamical field with dynamics determined by scale-invariant Weyl gravity, and is similar in spirit to an earlier proposal based on the Weyl-Dirac-Yang-Mills action [15, 16]. The quantum theory of the conformally invariant Weyl action has a number of remarkable features which are desirable in a fundamental theory for quantum gravity. For example, it is known to reduce to Einstein gravity in the low energy limit [14] and there are indications that it may be renormalizable [17, 15] and asymptotically free [18]. It is intriguing that a nonconformal theory such as Born-Infeld theory can in fact be coupled to gravity in a conformally invariant manner.

2 Born-Infeld and Brane Actions

The Nambu-Goto action for a p -brane with $p = n - 1$ is

$$S_{NG} = -T_p \int d^n \sigma \sqrt{-\det(G_{\mu\nu})}, \quad (1)$$

where T_p is the p -brane tension and

$$G_{\mu\nu} = G_{ij} \partial_\mu X^i \partial_\nu X^j \quad (2)$$

is the world-volume metric induced by the spacetime metric G_{ij} . The non-linear form of the action (1) is inconvenient for many purposes, including that of quantisation. However, introducing an intrinsic worldvolume metric $g_{\mu\nu}$ allows one to write down the equivalent action [19, 20, 21]

$$S_P = -\frac{1}{2} T'_p \int d^n \sigma \sqrt{-g} [g^{\mu\nu} G_{\mu\nu} - (n-2)\Lambda], \quad (3)$$

where $g \equiv \det(g_{\mu\nu})$ and Λ is a constant. The metric $g_{\mu\nu}$ is an auxiliary field which can be eliminated using its equation of motion to recover action (1). The constants T_p and T'_p are related by

$$T'_p = \Lambda^{\frac{n}{2}-1} T_p. \quad (4)$$

This form of the action is much more convenient than (1), as it is quadratic in ∂X . In particular, for strings ($n = 2$), Polyakov [19] showed that the functional integral over all closed compact worldsheets can be reduced to the quantum theory of the two-dimensional Liouville Lagrangian. However, the explicit evaluation of the functional integrals [19] relied on the fact, special to two dimensions, that it is possible to change coordinates in such a way that the worldsheet metric becomes conformally Euclidean. Thus the integration over X yields an integral which depends on the conformal factor only through the conformal anomaly.

The Born-Infeld action for a vector field A_μ in an n -dimensional space-time with metric $G_{\mu\nu}$ is

$$S_{BI} = -T_p \int d^n \sigma \sqrt{-\det(G_{\mu\nu} + F_{\mu\nu})}, \quad (5)$$

where $F = dA$ is the Maxwell field strength. A related $(n-1)$ -brane action is

$$S_{DBI} = -T_p \int d^n \sigma \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})}, \quad (6)$$

where $G_{\mu\nu}$ is the induced metric (2) and $\mathcal{F}_{\mu\nu}$ is the antisymmetric tensor field

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - B_{\mu\nu} \quad (7)$$

with $B_{\mu\nu}$ the pull-back of a space-time 2-form gauge field B ,

$$B_{\mu\nu} = B_{ij} \partial_\mu X^i \partial_\nu X^j. \quad (8)$$

The action (6) is closely related to the D-brane action [24, 25, 26, 27] and the Born-Infeld action (5) can be thought of as a special case of this, but with a different interpretation of $G_{\mu\nu}$. Just as in the case of the action (1), the non-linearity of (6) makes it rather difficult to study. However, a nondeterminantal action which is the analog of (3) for this case has been proposed in [12], based on the introduction of an auxiliary world-volume tensor field $\gamma_{\mu\nu}$ with both a symmetric part $\gamma_{(\mu\nu)}$ and an antisymmetric part $\gamma_{[\mu\nu]}$. Such ‘metrics’ have been used in alternative theories of gravitation; see e. g. [28, 29]. The action which is classically equivalent to (6) is

$$S' = -\frac{1}{2}T'_p \int d^n\sigma \sqrt{-\gamma} \left[(\gamma^{-1})^{\mu\nu} (G_{\mu\nu} + \mathcal{F}_{\mu\nu}) - (n-2)\Lambda \right], \quad (9)$$

where $\gamma \equiv \det(\gamma_{\mu\nu})$; the inverse tensor $(\gamma^{-1})^{\mu\nu}$ satisfies

$$(\gamma^{-1})^{\mu\nu} \gamma_{\nu\rho} = \delta^\mu{}_\rho. \quad (10)$$

For $n \neq 2$, the $\gamma_{\mu\nu}$ field equation implies

$$G_{\mu\nu} + \mathcal{F}_{\mu\nu} = \Lambda \gamma_{\nu\mu} \quad (11)$$

and substituting back into (9) yields the Born-Infeld-type action (6) where the constants T_p, T'_p are related as in eq. (4). For $n = 2$, the action (9) is invariant under the generalised Weyl transformation

$$\gamma_{\mu\nu} \rightarrow \omega(\sigma) \gamma_{\mu\nu} \quad (12)$$

and the $\gamma_{\mu\nu}$ field equation implies

$$G_{\mu\nu} + \mathcal{F}_{\mu\nu} = \Omega \gamma_{\nu\mu} \quad (13)$$

for some conformal factor Ω .

The action (9) is *linear* in $(G_{\mu\nu} + \mathcal{F}_{\mu\nu})$ and so is much easier to analyse than (6). In particular, it is linear in F and quadratic in ∂X .

As was shown in [13], the action (6) can also be rewritten in a form which is quadratic in the field strength F , and is therefore simpler to analyse and quantise. This form of the action uses an intrinsic worldvolume metric $g_{\mu\nu}$ (for the remaining part of this section, we write $g_{\mu\nu} \equiv \gamma_{(\mu\nu)}$ with $\gamma_{[\mu\nu]} = 0$), just like (3). The action which is classically equivalent to (6) and is quadratic in the gauge field strength $F_{\mu\nu}$ is

$$S'' = -T''_p \int d^{p+1}\sigma (-G)^{\frac{1}{4}} (-g)^{\frac{1}{4}} [g^{\mu\nu} (G_{\mu\nu} - G^{\rho\sigma} \mathcal{F}_{\mu\rho} \mathcal{F}_{\sigma\nu}) - (p-3)\Lambda], \quad (14)$$

where $g \equiv \det(g_{\mu\nu})$ and Λ is a constant. For $p \neq 3$, the $g_{\mu\nu}$ field equation implies

$$g_{\mu\nu} = \frac{1}{\Lambda} (G_{\mu\nu} - G^{\rho\sigma} \mathcal{F}_{\mu\rho} \mathcal{F}_{\sigma\nu}) \quad (15)$$

and substituting back into (14) yields action (6). The constants T_p , T_p'' are related by

$$T_p'' = \frac{1}{4} \Lambda^{\frac{p-3}{4}} T_p. \quad (16)$$

For $p = 3$, the four-dimensional action (14) is invariant under the usual Weyl transformation

$$g_{\mu\nu} \rightarrow \omega(\sigma) g_{\mu\nu} \quad (17)$$

and the $g_{\mu\nu}$ field equation implies

$$g_{\mu\nu} = \Omega (G_{\mu\nu} - G^{\rho\sigma} \mathcal{F}_{\mu\rho} \mathcal{F}_{\sigma\nu}) \quad (18)$$

for some conformal factor Ω .

The actions (9) and (14) are easily generalised to the Dp -brane kinetic term

$$S = -T_p \int d^{p+1} \sigma e^{-\phi} \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} \quad (19)$$

where ϕ , $G_{\mu\nu}$ and $B_{\mu\nu}$ are the pullbacks to the worldvolume of the background dilaton, background metric and background NS antisymmetric two-form fields, $F = dA$, with A the $U(1)$ world-volume gauge field, and $\mathcal{F}_{\mu\nu}$ is defined as in (7). This action gives the effective dynamics of the zero-modes of the open strings with ends tethered on a D-brane when F is slowly varying, so that corrections involving ∇F can be ignored, and has therefore played a central role in recent studies of D-brane dynamics and string theory duality [30].

As shown in [12, 13], the methods above can also be applied to the kappa-symmetric action for a D-brane [31, 32, 33], to the low energy effective action for an open type I string [34], to static-gauge Dp -brane actions [35], and to the M-theory five-brane action [36, 37]. For example, a classically equivalent form of the latter which is quadratic in the field strength of the worldvolume self-dual two-form tensor gauge field was given in [13] using a (symmetric) auxiliary worldvolume metric.

3 Brane Effective Actions

Whatever the nature of the fundamental theory at short distances, the space-time effective action at large distances in the presence of a brane (which can be a D-brane, an M-brane or a more general domain wall) is of the form

$$S = S_{bulk} + S_{brane}. \quad (20)$$

Here the action

$$S_{bulk} = \int d^D X (-G)^{\frac{1}{2}} \left[-\Lambda + 2M^{D-2} R(G) + \dots \right], \quad (21)$$

where M is the D -dimensional Planck mass and Λ the cosmological constant, describes the bulk gravitational and other massless fields. In string theory, S_{bulk} is the bosonic part of the appropriate low-energy supergravity theory, e. g. type IIA or type IIB supergravity in the Einstein frame; in M-theory, it is the bosonic part of eleven dimensional supergravity [38]. In both cases, S_{bulk} receives higher curvature corrections and also includes other terms for the remaining massless spacetime fields; this is indicated by the ellipsis in (21). The action S_{brane} describes the effective dynamics of the massless bosonic modes on the brane worldvolume. For example, the bosonic part of the effective world-volume action for a D-brane in a type II supergravity background contains the terms [39, 40, 41]

$$S_{Dbrane} = -T_p \int d^n \sigma e^{-\phi} \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} + T_p \int_{W_n} C e^{\mathcal{F}} \left(\frac{\hat{\mathcal{A}}(R_T)}{\hat{\mathcal{A}}(R_N)} \right)^{\frac{1}{2}}. \quad (22)$$

The second term is a Wess-Zumino term and gives the coupling to the background Ramond-Ramond r -form gauge fields $C^{(r)}$ (where r is odd for type IIA and even for type IIB) as well as the gravitational curvature effects induced from the bulk geometry in which the brane is embedded. The potentials $C^{(r)}$ for $r > 4$ are the duals of the potentials $C^{(8-r)}$. In (22), C is the formal sum [40]

$$C \equiv \sum_{r=0}^9 C^{(r)}, \quad (23)$$

all forms in space-time are pulled back to the worldvolume of the brane W_n and it is understood that the n -form part of $C e^{\mathcal{F}}$, which is $C^{(n)} + C^{(n-2)}\mathcal{F} + \frac{1}{2}C^{(n-4)}\mathcal{F}^2 + \dots$, is selected. In (22), $\hat{\mathcal{A}}$ denotes the Dirac ‘roof’ genus whose square root has an expansion in powers of the curvature two-form, and the components of the curvature are split: R_T denotes components with tangent-space indices, while R_N denotes components in the normal bundle (see [41] for notational details).

The first term in (22) receives corrections involving derivatives of the field strength F [34], as well as gravitational curvature contributions induced by the background geometry or by non-trivial worldvolume embeddings [42].

Introducing $\gamma_{\mu\nu}$ as before, we obtain the classically equivalent D-brane action

$$\begin{aligned} S''_{Dbrane} = & -T_p'' \int d^{p+1} \sigma (-G)^{\frac{1}{4}} (-g)^{\frac{1}{4}} e^{-\phi} [g^{\mu\nu} (G_{\mu\nu} - G^{\rho\sigma} \mathcal{F}_{\mu\rho} \mathcal{F}_{\sigma\nu}) - (p-3)\Lambda] \\ & + T_p \int_{W_n} C e^{\mathcal{F}} \left(\frac{\hat{\mathcal{A}}(R_T)}{\hat{\mathcal{A}}(R_N)} \right)^{\frac{1}{2}}. \end{aligned} \quad (24)$$

The field equation for $g_{\mu\nu}$ is given in (15) and (18); substituting back into (24) yields (22).

4 Dynamical Gravity on the Brane

In previous sections, the worldvolume tensor fields $\gamma_{\mu\nu}$ were merely auxiliary, i. e. they were not dynamical. Motivated by the remarks in the introduction, we now propose to promote such fields to dynamical gravitational ‘metric’ potentials on the world volume. The new actions presented below are also of some interest in their own right, as they describe the dynamics of nonlinear gauge fields interacting non-trivially with gravity. In certain limits, the worldvolume dynamics of the gauge fields is of Born-Infeld type, while the dynamics of the intrinsic ‘metric’ $\gamma_{\mu\nu}$ is governed by Einstein gravity or an appropriate (symmetric) scale-invariant or a nonsymmetric generalisation thereof. In the case where the worldvolume tensor introduced is a symmetric metric $g_{\mu\nu}$, we also propose an action which specifies the dynamics of gauge fields coupled to *two* worldvolume metrics, namely $g_{\mu\nu}$ and the induced metric $G_{\mu\nu}$. In the particular case of the 3-brane, the Weyl invariance with respect to rescalings of $g_{\mu\nu}$ suggests that the dynamics of this metric should be governed by a scale-invariant Weyl term.

We begin with the case of a symmetric intrinsic metric $g_{\mu\nu}$. We first consider dynamical induced gravity on the worldvolume, i. e. we add an Einstein-Hilbert term as well as cosmological and higher derivative terms to (14). Thus we consider the action

$$\begin{aligned} S''_{brane} = & -T_p'' \int d^{p+1} \sigma (-G)^{\frac{1}{4}} (-g)^{\frac{1}{4}} [g^{\mu\nu} (G_{\mu\nu} - G^{\rho\sigma} \mathcal{F}_{\mu\rho} \mathcal{F}_{\sigma\nu}) - (p-3)\Lambda_1] \\ & + \lambda \int d^{p+1} \sigma (-G)^{\frac{1}{2}} [\Lambda_2 + aR(G) + bR^2(G) + \dots]. \end{aligned} \quad (25)$$

Here $R(G)$ denotes the scalar curvature of the induced metric $G_{\mu\nu}$ and λ is some coupling. Note that we have allowed the possibility of having a cosmological constant on the world-volume which differs from that of the bulk theory. This type of action does actually arise in string theory. For example, the $R^2(G)$ and other quadratic in curvature corrections to the Dirac-Born-Infeld part of the effective action for a single Dp-brane were obtained in [42] to lowest order in the open string loop expansion.

In (25), the intrinsic metric is auxiliary and the worldvolume gauge fields have Born-Infeld-type dynamics. Consider instead a dynamical intrinsic metric $g_{\mu\nu}$. Adding an Einstein-Hilbert and a cosmological term to (14) gives

$$\begin{aligned} S''_{brane} = & -T_p'' \int d^{p+1} \sigma (-G)^{\frac{1}{4}} (-g)^{\frac{1}{4}} [g^{\mu\nu} (G_{\mu\nu} - G^{\rho\sigma} F_{\mu\rho} F_{\sigma\nu}) - (p-3)\Lambda_1] \\ & + \lambda \int d^{p+1} \sigma (-g)^{\frac{1}{2}} [\Lambda_2 + aR(g)] \end{aligned} \quad (26)$$

where $R(g)$ denotes the scalar curvature of the intrinsic metric $g_{\mu\nu}$ and λ is some coupling. Moreover, quadratic or higher curvature corrections $X_{\mu\nu}$ can be introduced via a coupling $\sqrt{-g}g^{\mu\nu}X_{\mu\nu}$. Although this is more natural in the present

nondeterminantal framework, it is also possible to replace the second line in (26) with a determinant gravity action of the form $\int d^n \sigma \det[ag_{\mu\nu} + bR_{\mu\nu} + cX_{\mu\nu}]$; see [23] for a discussion of physical requirements on such actions.

The case of a 3-brane is rather special as a result of the Weyl invariance (17). Clearly this symmetry will not be preserved in the model described by (26), because Einstein gravity is not Weyl invariant². However it is possible to couple (6) to gravity in a Weyl invariant manner by adding a (fourth derivative) Weyl term,

$$S''_{brane} = -T_3'' \int d^{p+1} \sigma (-G)^{\frac{1}{4}} (-g)^{\frac{1}{4}} g^{\mu\nu} (G_{\mu\nu} - G^{\rho\sigma} \mathcal{F}_{\mu\rho} \mathcal{F}_{\sigma\nu}) \\ + \lambda \int d^{p+1} \sigma (-g)^{\frac{1}{2}} C^\mu{}_{\nu\rho\sigma}(g) C^\nu{}_{\mu\lambda\tau}(g) g^{\lambda\rho} g^{\tau\sigma} \quad (27)$$

where $C^\mu{}_{\nu\lambda\rho}$ denotes the four-dimensional Weyl conformal tensor of $g_{\mu\nu}$,

$$C^\mu{}_{\nu\lambda\rho} \equiv R^\mu{}_{\nu\lambda\rho} - \frac{1}{2} \left(\delta^\mu_\lambda R_{\nu\rho} - \delta^\mu_\rho R_{\nu\lambda} - g_{\nu\lambda} R^\mu{}_\rho + g_{\nu\rho} R^\mu{}_\lambda \right) \\ + \frac{1}{6} R \left(\delta^\mu_\nu g_{\lambda\rho} - \delta^\mu_\rho g_{\nu\lambda} \right). \quad (28)$$

By construction, this is invariant under the transformation (17).

As explained above, we can also couple the gauge fields covariantly to both the intrinsic metric $g_{\mu\nu}$ and the induced metric $G_{\mu\nu}$ on the p -brane via the action

$$S''_{brane} = -T_p'' \int d^{p+1} \sigma (-G)^{\frac{1}{4}} (-g)^{\frac{1}{4}} [g^{\mu\nu} (G_{\mu\nu} - G^{\rho\sigma} \mathcal{F}_{\mu\rho} \mathcal{F}_{\sigma\nu}) - (p-3)\Lambda_1] \\ + \lambda \int d^{p+1} \sigma (-g)^{\frac{1}{4}} (-G)^{\frac{1}{4}} [\Lambda_2 + R(g)R(G)] \quad (29)$$

where $R(G)$ is the scalar curvature of the induced metric $G_{\mu\nu}$. Observe that the factors of the determinants ensure that the second line also transforms as a scalar under worldvolume diffeomorphisms.

Now consider the action (9), in which the intrinsic ‘metric’ $\gamma_{\mu\nu}$ has both a symmetric part $g_{\mu\nu}$ and an antisymmetric part $\gamma_{[\mu\nu]}$. We can promote this wordvolume tensor field to dynamical gravitational fields by adding terms which have previously been studied in the context of nonsymmetric gravitational theories, e. g. in [29]. Thus we consider the action

$$S'_{brane} = -\frac{1}{2} T_p' \int d^n \sigma \sqrt{-\gamma} \left[(\gamma^{-1})^{\mu\nu} (G_{\mu\nu} + \mathcal{F}_{\mu\nu}) - (n-2)\Lambda_1 \right] \\ + \lambda \int d^n \sigma L \quad (30)$$

where ω is an arbitrary coupling, and the Lagrangian density of the non-Riemannian geometry is given by

$$L = \sqrt{-\gamma} \gamma^{\mu\nu} R_{\mu\nu}(W) - 2\Lambda_1 \sqrt{-\gamma} - \frac{1}{4} \Lambda_2 \sqrt{-\gamma} \gamma^{\mu\nu} \gamma_{[\nu\mu]} - \frac{1}{6} \gamma^{\mu\nu} W_\mu W_\nu \quad (31)$$

²Einstein gravity can be made locally Weyl invariant using a scalar compensating field [22]. This procedure can be adapted to make a certain modification of Born-Infeld theory Weyl invariant, as discussed in [23].

Note the presence of the second cosmological constant Λ_2 associated with $\gamma_{[\mu\nu]}$. The curvature tensor $R_{\mu\nu}(W)$ is defined in terms of unconstrained nonsymmetric connections

$$W_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda - \frac{2}{3}\delta_\mu^\lambda W_\nu \quad (32)$$

$$W_\mu = W_{[\mu\lambda]}^\lambda \quad (33)$$

via the expression

$$R_{\mu\nu}(W) = \partial_\rho W_{\mu\nu}^\rho - \frac{1}{2}(\partial_\nu W_{\mu\rho}^\rho + \partial_\mu W_{\nu\rho}^\rho) - W_{\sigma\nu}^\rho W_{\mu\rho}^\sigma + W_{\sigma\rho}^\rho W_{\mu\nu}^\sigma \quad (34)$$

The field equations for $\gamma_{\mu\nu}$, $G_{\mu\nu}$ and $F_{\mu\nu}$ can be straightforwardly derived from the actions (25), (26), (27), (29) and (30). We expect these systems of equations to have a rich set of interesting solutions, including worldvolume instantons, black holes, extended solitons and brane world cosmologies.

5 Discussion

In this paper, motivated by recent ideas on brane world models and especially warped compactification and their relation to the AdS/CFT correspondence, we have considered various generalisations of the nondeterminantal forms of the Born-Infeld and related brane actions found in [12, 13] which involve dynamical gravity on the brane worldvolume. In such actions, the gauge fields couple both to a background or induced metric $G_{\mu\nu}$ and to an intrinsic tensor $\gamma_{\mu\nu}$. The latter can either be taken to be symmetric, in which case the brane kinetic terms are quadratic in the field strength F of the abelian gauge fields, or to have both a symmetric and an antisymmetric part, in which case the brane kinetic terms are linear in F . Either $G_{\mu\nu}$ or $\gamma_{\mu\nu}$ can be promoted to dynamical gravitational fields on the brane worldvolume by adding suitable gravitational contributions to the brane kinetic terms. Dynamical induced gravity occurs in string theory with D-branes, as seen e. g. in refs. [41, 42]. Dynamical intrinsic gravity does not occur in this context, because the spectra of zero modes of the open strings tethered on D-branes do not include gravitons. However, it may turn out to have applications to brane world models in which a conformal or nonconformal quantum field theory on the brane is coupled to gravitational modes confined to the brane, as in [7].

If a general intrinsic tensor $\gamma_{\mu\nu}$ is chosen, then it is natural to let its worldvolume dynamics be given by the nonsymmetric gravitational theory [29]. This leads to interesting brane world versions of the cosmological scenarios discussed in [43]. For example, if the skew part of $\gamma_{\mu\nu}$ is assumed to be small, then the nonsymmetric gravitational field equations can be expanded about the Friedmann-Robertson-Walker

metric and the equation for the brane cosmological factor determines corrections due to a non-zero skew part $\gamma_{[\mu\nu]}$ and its derivatives [43]. It would be interesting to study such brane world models further.

If a symmetric intrinsic metric $g_{\mu\nu}$ is used instead, then it is more natural to consider Einstein gravity or a higher derivative symmetric gravitational theory on the brane. In the special case of 3-branes, the preservation of the generalised classical Weyl invariance (17) suggests that worldvolume gravity should be of the Weyl type, as in action (27). This describes the classical scale-invariant dynamics of a fluctuating brane world in which nonlinear abelian gauge fields couple to Weyl gravity. Quantum mechanically, the Weyl invariance (17) will be anomalous, both in the term quadratic in F and in the C^2 Weyl term [44]. It would be very interesting to compute the conformal anomaly for brane world models based on this action. Since (27) can easily be generalised to include an arbitrary number of abelian vectors and scalars conformally coupled to Weyl gravity, it may be possible to cancel the conformal anomaly. This would generalise the critical dimension of string theory in the formulation of Polyakov, and may yield a consistent theory of dynamical 3-branes. Similar ideas were recently proposed in [45] on the basis of the AdS/CFT correspondence.

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